| FULL LEGAL NAME | **LOCATION (COUNTRY)** | **EMAIL ADDRESS** | **MARK X FOR ANY NON-CONTRIBUTING MEMBER** |
| --- | --- | --- | --- |
| Shubhankit Singh | India | shubhubits313@gmail.com |  |
| Ebenezer Yeboah | Ghana | ebenezeryeboah46@gmail.com |  |
| Olawale Omoniyi Akande | Nigeria | waleomoniyi@hotmail.com |  |

| **Statement of integrity:** By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an “X” above). | |
| --- | --- |
| **Team member 1** | **Shubhankit Singh** |
| **Team member 2** | Ebenezer Yeboah |
| **Team member 3** |  |

| Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed.  **Note:** You may be required to provide proof of your outreach to non-contributing members upon request. |
| --- |
|  |

1. **a. Pricing European Call and Put Options Using Black-Scholes Closed Form**

The BSF(Black-Scholes) model is used here to calculate the prices of European call and put options with the following parameters: Underlying Asset(S) of $100, Strike price (K) of $100, time to maturity (T) of 0.25 years, risk-free rate (r) of 5%, and volatility (σ) of 20%.

* The calculated call option price (C) is $4.61.
* The calculated put option price (P) is $3.37.

**b. Compute the Greek Delta for the European Call and Put at Time 0 : (Here, time 0 is not the time to maturity)**

* The Delta for the call option is 0.57. For every $1 increase in the underlying asset price, the call option's price is expected to increase by close to $0.57.
* The Delta for the put option is -0.43. For every $1 increase in the underlying asset price, the put option's price is expected to decrease by approximately $0.43.
* The positive Delta for the call option tells us that call options typically increase in value as the underlying asset's price increases. Conversely, the negative Delta for the put option means that put options usually increase in value as the underlying asset's price decreases.

**c. Sensitivity of Option Price w.r.t Volatility (Vega)**

* Vega, for both call and put options, is determined to be 19.64. This implies that a 1% increase in the underlying asset's volatility should see an increase in price for both call and put options by approximately $19.64.
* The prices of both options rose by around $0.99 when volatility increased from 20% to 25% (a five percent rise):
* For example, the price of a call option increased from $4.61 to $5.60. Also, The price of a put option went up from $3.37 to $4.36.
* This means that the change in pricing for these two types of contracts was similar because of a greater chance at expiration if implied volatility is high since it will add some value to the contract.
* In addition, this movement in vega with respect to changing volatilities affects both calls and puts symmetrically because they have the same vega measure within the Black-Scholes framework. This symmetry comes about as there is no bias towards any direction concerning pricing outcomes in BSM’s log-normal assumption about return distribution, as prices can move equally in either direction without being influenced by volatility along them.
* Nevertheless, actual effects of changes in volatility on call and put options are likely different due to market conditions such as demand/supply dynamics varying or other factors like macro-economy divergence differential may cause differences between the two kinds of derivatives.

1. **5a.** The number of steps used is 100,000 and it helps achieve a more accurate answer.

**5b**. The code uses a Monte Carlo simulation to value European call and put options as well as calculating delta and vega.

* The simulation generates random normal variables for each simulation run. It then uses these variables to simulate stock price paths based on the geometric Brownian motion (GBM) process.
* Once the stock price paths are simulated, the code calculate the payoffs for both the call and put options at maturity. It computes the average payoffs and discounts them back to present value using the risk-free interest rate to estimate the option prices.
* Delta measures the sensitivity of the option price to changes in the underlying asset price. The code approximates delta by computing the change in option price with a small increment in the underlying stock price.
* Vega measures the sensitivity of the option price to changes in the volatility of the underlying asset. The code approximates vega by computing the change in option price with a small increment in the volatility parameter.

The number of simulations chosen for the Monte Carlo simulation affects the accuracy and precision of the estimated option prices and sensitivities. A more significant number of simulations generally leads to more accurate estimates but also requires more computational resources. In this case, the number of simulations is 100,000, which balances accuracy and computational efficiency. Increasing the number of simulations can refine the estimates but may increase computational time.

**6a.** For the European call option, delta measures the change in the call option price with respect to a slight change in the underlying stock price.

For the European put option, delta measures the change in the put option price with respect to a slight change in the underlying stock price.

The European call option typically has a higher absolute delta than the European put option when both options are at the money (ATM). This means that the call option is more sensitive to underlying stock price changes than the put option.

Delta values may vary based on factors such as the strike price, time to expiration, and volatility, but in general, the call option tends to have a higher delta than the put option when both options are ATM.

**6b.** Delta for European call options is typically positive, indicating that the call option price increases when the underlying asset price rises.

Delta for European put options is typically negative, suggesting that the put option price decreases as the underlying asset price rises.

Delta is a proxy for an option's price change concerning changes in the underlying asset's price.

A positive delta implies that the option price increases when the underlying asset price increases, and vice versa for a negative delta.

A positive delta for call options reflects the bullish nature of call options. The call option becomes more valuable as the underlying asset price rises, leading to a positive delta.

A negative delta for put options reflects the inverse relationship between put option prices and the underlying asset price. As the underlying asset price rises, the put option loses value, resulting in a negative delta.

**7a.** Option prices increase with higher volatility, reflecting the increased uncertainty and potential for larger price movements in the underlying asset.

**7b.**  Asymmetric payoff structures and the effects of volatility on option pricing can potentially impact the difference in the increase of volatility. Call options benefit from increases in volatility because higher volatility increases the likelihood of large price movements in the underlying asset, which can lead to higher potential gains for call option holders. As volatility increases, the probability of the underlying asset surpassing the strike price also increases, resulting in higher call option prices. Therefore, a 5% increase in volatility generally leads to a more significant increase in call option prices.

Put options also benefit from increases in volatility, but to a lesser extent compared to call options. This is because put options profit from declines in the underlying asset's price, and higher volatility increases the likelihood of larger downward price movements. However, put options may experience diminishing returns with increasing volatility because there is a limit to how low the underlying asset's price can go (i.e., it cannot fall below zero). Therefore, while put option prices generally increase with higher volatility, the magnitude of the increase may be smaller compared to call options. The impact of a 5% increase in volatility on put option prices may be less pronounced than on call option prices.

1. **(A) Put-Call Parity**Put-call parity is satisfied under both the Black-Scholes option pricing method and the Monte-Carlo Simulation method. This is confirmed from the result of our put-call parity check for both methods applied.  **(B)**The prices obtained under the Black-Scholes Method and the Monte-Carlo Simulation method converge over a large enough number of simulations. As the number of simulations (sample size) in Monte Carlo increases, the simulated prices should converge to the theoretical Black-Scholes prices. This is due to the law of large numbers and the central limit theorem. The law of large numbers states that as the number of independent, identically distributed random variables (simulations in this case) increases, the sample mean (average) converges to the expected value. Also, the central limit theorem says thatdistribution of the sum (or average) of a large number of independent, identically distributed random variables approaches a normal distribution, regardless of the shape of the original distribution.
2. **Price American Option Call using monte-carlo and GBM :**

The Monte Carlo Method using GBM was used to calculate the prices of American call option with the following parameters: underlying asset price (S) of $100, strike price (K) of $100, time to maturity (T) of 0.25 years, risk-free rate (r) of 5%, and volatility (σ) of 20%.

**The calculated American call option price (C) is $80.6**

**B.** **Delta Sensitivity Analysis: -**

**Calculation: Delta of the Call Option = 0.5.**

Delta measures the sensitivity of an option’s price to a $1 change in the price of the underlying asset. A Delta is said to be 0.5 if for every $1 increase in the underlying stock price, the call option price will rise by $0.50; this is usually seen with at-the-money options where the probability of ending in or out of the money is equal.

**Meaning of Positive Delta:** It means that when delta is positive for a call option, its price increases as well when stock prices go up hence making sense because calls give right to buying stocks at strike prices which become more valuable as their market value increases.

**c. Volatility Sensitivity (Vega) Analysis:**

**Calculation**: **The Vega value provided is 947.58.**

**Interpretation:** Vega measures the sensitivity of the option's price to a 1% change in the underlying asset's implied volatility. A Vega of 947.58 means that for every 1% increase in volatility, the call option's price will increase by $947.58. This is a significant figure and indicates high sensitivity to volatility.

**Impact of Volatility Change:** From 20% to 25% (a 5% increase): The option price will increase by Vega multiplied by the percentage change in volatility.

**Calculation with Given Vega:** New Call Price = Initial Call Price + (Vega × %age Increase in Volatility = $80.6 + (947.58 × 5%) = $80.6 + $47.379 ≈ $127.979

**Impact:** This change represents the additional premium the option buyer will pay to protect against the increased uncertainty (volatility) in the underlying asset's price movement.

**c. Volatility Impact:**

* The option is very volatile when the call option price leaps up. This is the intrinsic value of most options in such a state.
* As volatility increases, so do option prices, leading to an anticipated increase in an option's chances of expiring in the money. This is often true for American options since volatility tends to rise and fall sharply, which might offer higher returns if one chooses wisely, as it becomes better off exercise early.
* To sum up; Delta of 0.5 shows a moderate sensitivity to changes in stock prices, while Vega of 947.58 shows a significant sensitivity to changes in volatility of the American Call option. The provided calculations show how much the option's price should change due to underlying stock price changes and volatility adjustments.



**5a.** 100,000 simulations are chosen to help achieve reliable estimates by making them more accurate.

**5b. Monte Carlo Simulation**

●The simulation generates random normal variables to simulate daily stock price movements over the specified time horizon.

●For each simulation, the stock price is simulated using the geometric Brownian motion (GBM) process, which incorporates the risk-free rate, volatility, and time step.

●At each time step, the stock price is updated based on the GBM equation, ensuring it remains non-negative.

●After simulating multiple paths of stock prices, the option payoffs at maturity are calculated based on the difference between the strike price and the simulated stock prices.

●The option price is estimated by taking the mean of the discounted option payoffs across all simulations.

**Delta Computation**

●Delta measures the sensitivity of the option price to changes in the underlying asset price. Here, it's estimated by incrementally changing the stock price and observing the resulting change in the option price.

●Delta for the American put option is computed using finite differences, where a small increment in the stock price is introduced, and the change in the option price is divided by the increment.

**Vega Computation**

●Vega measures the sensitivity of the option price to changes in volatility. It is estimated by incrementally changing the volatility parameter and observing the resulting change in the option price.

●Vega for the American put option is computed by introducing a small volatility parameter increase and observing the option price change.

**6a**. Delta measures the sensitivity of the option price to changes in the underlying asset price. It indicates the expected change in the option price for a a unit change in the underlying asset price.

**6b.**  The delta of a call option is positive, indicating that the option price increases with an increase in the underlying asset price.

**7a.**  The change in option prices due to the increase in volatility provides insights into the vega, which measures the sensitivity of option prices to changes in volatility. If the prices increase with higher volatility, the options have positive vega, indicating that their prices increase as volatility rises and if the prices decrease with higher volatility, the options have negative vega.

**7b.** With an increase in volatility, the potential profit from holding a call option also increases. This is because higher volatility leads to larger price movements in the underlying asset, increasing the likelihood of the asset's price rising above the strike price. Also, With an increase in volatility, the potential profit from holding a put option also increases, as higher volatility leads to larger price movements in the underlying asset, increasing the likelihood of the asset's price falling below the strike price.



| Strike Price | Call Price | Delta(Call) | Vega(Call) | Put | Delta (Put) | Vega (Put) |
| --- | --- | --- | --- | --- | --- | --- |
| 90 | 86.03 |  |  |  |  |  |
| 95 | 86.32 |  |  |  |  |  |
| 100 |  |  |  |  |  |  |
| 105 |  |  |  |  |  |  |
| 110 |  |  |  |  |  |  |



* Call option with 110% moneyness price: $10.9
* Call option delta: 0.87
* Put option with 95% moneyness price: $6.35
* Put option delta: -0.63
* Portfolio 1 delta (buying both the call and put): 0.24
* Portfolio 2 delta (buying the call and selling the put): 1.5
* These values suggest the following insights:
* Call Option (110% moneyness): The call option price is $10.9, and the delta of 0.87 indicates that for every dollar increase in the underlying asset's price, the call option cost is expected to increase by 87 cents.
* Put Option (95% moneyness): The put option is priced at $6.35. A negative delta of -0.63 implies that for every dollar increase in the underlying asset's price, the put option's price is expected to decrease by 63 cents.
* Portfolio 1 Delta: By purchasing both the call and put option, the portfolio delta is 0.24. This low delta value indicates that the portfolio is less sensitive to changes in the underlying asset's price. It suggests a nearly delta-neutral position, which might be part of a hedging strategy that benefits from changes in other factors like volatility rather than price movements.
* Portfolio 2 Delta: By buying the call and selling the put, the portfolio has a delta of 1.5. This is a leveraged position that would benefit from an upward movement in the price of the underlying asset. For every dollar increase in the underlying asset's price, the portfolio's value would increase by $1.5.

1. The estimated price of the Up-and-Out barrier option is an important metric for investors and traders, as it explains the market's perception of the option's value given current market conditions. The estimated price obtained from the Monte Carlo simulation gives an assessment of the option's value, considering various possible paths of the underlying asset's price movement.

With consideration of factors such as the strike price, barrier level, time to maturity, risk-free interest rate, and volatility, a higher estimated price suggests that the option is more valuable, while a lower estimated price indicates that the option may be less valuable, which could be due to lower volatility, shorter time to maturity, or a higher barrier level.

| **Option Type** | **Characteristics** | **Pricing Relationship** |
| --- | --- | --- |
| Vanilla Option | Has no barrier features | They are generally more expensive than UAI and UAO options |
| Up-and-In (UAI) Option | Barrier level, becomes active when crossed | Typically less expensive than vanilla options, more expensive than UAO options |
| Up-and-Out (UAO) Option | Barrier level, becomes worthless when crossed | Generally less expensive than both vanilla and UAI options |

In addition to the above, the following are addition comparisons among the listed option types:

* The barrier of an UAO acts as a risk mitigator, which makes the options less expensive than vanilla options due to the potential early termination.
* UAI poses a reduced risk for the option writer because it has an initial period when the option is inactive, which is why it is less expensive than the UAI.
* The price hierarchy is in the order: Vanilla > UAI > UAO.

| Q # | Type | Exer | GWP1  Method | GWP2  Method | GWP1 Price | GWP2  Price | %Diff |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | ATM Call | Eur | Binomial BS and Trinomial | BSF Closed form | $4.60 | $4.61 | (4.6-4.61)/4.6= -0.5% |
| 2 | ATM Put | Eur | Binomial BS and Trinomial | BSF Closed form | $3.36 | $3.37 | (3.36-3.37)/3.36=-0.5% |
| 3 | ATM Call | Amer | Binomial | MC GBM | $4.60(wrong) | $80.6 |  |
| 4 | ATM Put | Amer | Binomial | MC GBM | $3.48(wrong) | $22.54 |  |

**Works Cited**

“Conceptual explanation of the relationship between gamma and vega plotted against delta for a European call option.” *Quantitative Finance Stack Exchange*, 13 February 2019, https://quant.stackexchange.com/questions/44034/conceptual-explanation-of-the-relationship-between-gamma-and-vega-plotted-agains. Accessed 27 February 2024.

“Geometric Brownian motion.” *Wikipedia*, https://en.wikipedia.org/wiki/Geometric\_Brownian\_motion. Accessed 27 February 2024.

Kvilhaug, Suzanne. “Monte Carlo Simulation: History, How it Works, and 4 Key Steps.” *Investopedia*, https://www.investopedia.com/terms/m/montecarlosimulation.asp. Accessed 27 February 2024.

Mitchell, Cory, and Suzanne Kvilhaug. “Up-and-Out Option: What it is, How it Works, Example.” *Investopedia*, https://www.investopedia.com/terms/u/up-and-outoption.asp. Accessed 27 February 2024.

Scott, Gordon. “Black-Scholes Model: What It Is, How It Works, Options Formula.” *Investopedia*, https://www.investopedia.com/terms/b/blackscholes.asp. Accessed 27 February 2024.

“Up-And-Out Barrier Option Explained - moneyland.ch.” *Moneyland*, https://www.moneyland.ch/en/up-and-out-barrier-option-definition. Accessed 27 February 2024.

“Up-And-Out Barrier Option Explained - moneyland.ch.” *Moneyland*, https://www.moneyland.ch/en/up-and-out-barrier-option-definition. Accessed 27 February 2024.